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## **PHASE NOISE EFFECTS AND ITS COMPUTATIONALLY EFFICIENT REDUCTION TECHNIQUE IN OFDM SYSTEM**

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## **ABSTRACT**

Orthogonal frequency division multiplexing (OFDM) systems, is one of the future of high rate wireless communications, exhibit great sensitivity to the effects of phase noise and the time-varying propagation channel, which can introduce inter-channel interference (ICI) due to the loss of orthogonality among subcarriers and common phase error (CPE) which causes the rotation of constellation. The presence of phase noise can reduce the signal to noise ratio (SNR) at the receiver, and hence reduce the bit error rate (BER) and data rate.

There are various methods of phase noise mitigation already proposed in the literature but are of high complexity. In this paper a computationally efficient technique has been proposed, both by theory and computer simulations of the proposed scheme are shown which can effectively mitigate the effect caused by phase noise and improve the BER of OFDM systems.

**KEYWORDS:** Common phase error (CPE), inter-carrier interference (ICI), orthogonal frequency division multiplexing (OFDM), phase noise.

#### **INTRODUCTION**

OFDM can be viewed as either a multiplexing or modulation technique. In OFDM the signal is first split into independent channels, modulated by data and then multiplexed again to create the OFDM carrier. The sub-carriers are orthogonal to each other to improve spectral efficiency [4]. At the receiver side it is easy to recover data in each subcarrier till the carriers are orthogonal to each other. Larger the number of carriers gives better spectral efficiency. The main concept is the Orthogonality of the sub-carriers in OFDM system. Orthogonal Frequency Division Multiplexing (OFDM) is one of the most efficient multiplexing techniques to support the future wireless communication system [17]. It is because of its efficient bandwidth performance and high data rate. The affect like Inter Symbol interference (ISI) is also very less than the system compare to the other multiplexing techniques. Unfortunately OFDM is highly sensitive to the synchronization errors such as Carrier frequency offset, timing jitter and phase noise. The effect of CFO reduces the signal amplitude and result in interference between the carriers. Phase noise is caused due to the imperfection of the local oscillator (LO). The high spectral efficiency of OFDM is achieved due to partly overlapping spectra for the different subcarriers. When the system is perfectly synchronized, all the subcarriers are orthogonal but due to imperfect local oscillators (LO) at either transmitter (TX) or receiver (RX) side of the system carrier frequency offset or phase noise (PN) occurs, which result in loss of orthogonality between the subcarriers and intercarrier interference (ICI) occurs.

The influence of PN on the performance of an OFDM system has been published in several publications. [3]–[14]. They all show that the influence can be split into a multiplicative part, which is common to all subcarriers and therefore often referred to as common phase error (CPE) which causes rotation of signal constellation, and an additive part, which is often referred to as inter-carrier interference (ICI). Although the CPE is identified as the main performance limiting factor for coherent detection based receivers, CPE is more severe as it causes the constellation rotation. Various correction approaches for the CPE have been proposed previously [3], [5]. Some approaches for correction of both CPE and ICI have been proposed in [6], [7] but have a high complexity.

To compensate phase noise in OFDM system, various methods have been proposed in the literature and these methods can be divided into two approaches: [9] time domain and frequency domain

In the time domain approaches [15] the phase noise is removed before applying digital Fourier transform at the receiver, this can be done by extracting a single pilot sub-carrier signal from each OFDM symbol to drive a phase lock loop (PLL) for phase noise mitigation [9], [11], but these approaches need a special pilot pattern with pilots inserted in the middle of the bandwidth which is impossible in practice.

Frequency domain approaches requires processing after Digital Fourier Transform which corrects both CPE and **ICI.**  This is feasible in practice regardless of pilot patterns or channel environments. Hence, we generally study the performance of various methods in the frequency domain. A conventional frequency domain method compensates for CPE or its phase. However, this method has some drawback as it neglects ICI term, which is the important contribution of phase noise. Therefore, it is required to further suppress phase noise by considering both the CPE and ICI

#### **Phase Noise Model**

Phase noise is cause by imperfect local oscillator, for the sake of simplicity we always consider oscillator to be ideal in calculation [7], but in practice impulse response of practical oscillator has got some side bands which causes phase noise in the OFDM signal at the receiver. For the local oscillator at centre frequency  $f_c$ , the oscillator output at time instant t can be expressed as  $\exp\{i(2\pi f_c t + \Phi(t))\}$ , where  $\Phi(t)$  is the phase noise at the time instant t. The Phase noise Ф(t)*,* generated at both transmitter and receiver oscillators. Phase noise produced by a free running oscillator. It is usually modeled by a Wiener process, can be described as a continuous Brownian motion process [7] or a random Wiener process given by

$$
\Phi(t) = \int_{0}^{t} u(t)dt
$$
 (1)

It has zero mean and variance of  $2\pi \beta t$ , where  $\beta$  denotes the phase noise line width. Furthermore, it can be shown that  $E[{ \Phi(t) + \Phi(t+\tau)}^2] = 2\pi\beta |\tau|$  (2)

Phase noise has independent Gaussian increments and its power is a monotonically increasing function of time. This indicates that its power could be very large as time

#### **Ofdm System With Phase Noise**



Figure OFDM system model (receiver section) in the presence of phase noise

. A theoretical analysis of phase noise effects in OFDM system is found in [10][12][14]. The received OFDM signal corrupted by phase noise  $\Phi(n)$  is given as:

$$
Y_m(n) = (X_m(n) \otimes H_m(n) + W_m(n)) e^{j \Phi m(n)} \quad (4)
$$

Where  $X(n)$ ,  $H(n)$  and  $\Phi(n)$  denotes the sample of transmitted signal, impulse response of the channel and phase noise respectively.

After taking the discrete Fourier transform (DFT), [7] the information signal at the at the subcarrier k  $(k=0,1,2,\ldots,N-1)$  is given as

$$
Y_m(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} Y_m(n) e^{-j2\pi nk/N}
$$
  
= 
$$
\sum_{i=0}^{N-1} H_m(i) H_m(i) C_m(k-i) + W_m(k)
$$
 (5)

Where  $Hm(k)$ ,  $\mathcal{C}m(k)$  and  $Wm(k)$  are the DFT of the channel impulse response, phase noise and AWGN noise respectively.

The received symbol  $Ym(k)$  can be written as

$$
Y_m(k) = X_m(k)H_m(k)C_m(0) +
$$
  

$$
\sum_{\substack{i=0 \ i \neq k}}^{N-1} H_m(i)X_m(i)C_m(k-i) + W_m(k)
$$
 (6)

Where

$$
C_m(k) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\Phi m(n)} e^{-j2\pi n k/N}
$$

In order to separate the signal and noise terms, let us suppose that phase noise is small [3], so that:

$$
e^{j\Phi(n)} = 1 + j\Phi(n) \tag{7}
$$

Now the demultiplexed signal becomes

$$
y(k) \cong s_k + \frac{j}{N} \sum_{r=0}^{N-1} s_r \sum_{n=0}^{N-1} \Phi(n) \cdot e^{j\frac{2\pi}{N}(r-k)n}
$$
 (8)  

$$
y(k) \cong e_{k} + s_k
$$
 (9)

There is an error term  $e_k$  for each sub-carrier which is added to useful signal  $s_k$ .

#### **PHASE NOISE ESTIMATION AND MITIGATION**

OFDM suffers severe performance degradation in the presence of phase noise [1], [2]. Different methods have been proposed in the literature to correct phase noise either in the time domain or in the frequency domain [5-8]. The method proposed here is computationally efficient [16]. This method is applied for the case of only if the phase noise is very small. If we consider phase noise to be small then we have to estimate only CPE and consider inter carrier interference (ICI) as Gaussian noise. Directly estimating CPE saves computational complexity needed for extracting its phase from pilot signals, and results in an improved estimation accuracy and hence better receiver performance, but in case of large phase noise effects of both ICI and CPE should be eliminated [6], [13], [18].

Equation (6) can be rewritten as:

$$
y_m(k) = X_m(k)H(k) C_m(0) + \dot{n}_m(k) \tag{13}
$$

Where  $\dot{n}_m(k)$  contain AWGN noise and ICI term.

$$
\dot{n}_{m}(k) = \sum_{\substack{i=0 \ i \neq k}}^{N-1} H_{m}(i) X_{m}(i) C_{m}(k-i) + W_{m}(k) \quad (14)
$$

 $Y = X \cdot \hat{\theta} + e$ 

Equation (13) can be rewritten as:

From the above equation (13) 'e' can be calculated as  $e = Y - X \hat{\theta}$ 

To determine the least square estimator we write sum of square of the residuals (as a function of  $\hat{\theta}$ ) as:

$$
S(\hat{\theta}) = \sum e_i^2 = e^T e
$$

$$
= (Y - X.\hat{\theta})^T (Y - X.\hat{\theta})
$$

$$
= Y^T Y - Y^T \hat{\theta} - \hat{\theta}^T X^T Y + \hat{\theta}^T X^T X \hat{\theta} \qquad (15)
$$

The least square estimator is obtained by minimizing  $S(\hat{\theta})$ 

Now to get the estimate that gives the least square error the above equation (15) w.r.t  $\hat{\theta}$  and equate to zero

$$
\frac{\delta S}{\delta \hat{\theta}} = -2X^T Y + 2X^T X \hat{\theta} = 0
$$

$$
\hat{\theta} = (X^T X)^T (X^T Y) \tag{16}
$$

Where  $X$  represents the set of pilot symbol inserted at the transmitter, operator  $T$  denotes hermitian transpose (conjugate transpose).

The detected data bit  $\dot{x}(k)$  can be obtained from the CPE-corrected signal

$$
\dot{x}(k) = \left(\hat{\theta}\right)^{-1} Y_m(k) \tag{17}
$$

#### **SIMULATION RESULTS**

The OFDM model used in this simulation is IEEE.802.11a (WLAN)

Addition of phase noise degrade the performance of OFDM sysytem it is shown in fig (1) ,that addition of phase noise at a phase noise variance of 1.55 increases the bit error rate. Fig (2) shows the scater plot of 16 QAM with out phase noise addition and in obsence of AWGN noise,it shows no rotation in the constilation, but with phase noise addition rotation of constilation takes place which is shown in fig (3).



*Fig.2. BER vs SNR plot at a phase noise variance of 1.55*

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*Fig.3. scatter plot of 16 QAM without phase noise and without AWGN noise.*



*Fig.4. scatter plot of 16 QAM with out phase noise but with AWGN noise*



*Fig.5. scatter plot of 16 QAM with phase noise at a phase noise variance of 1.55*



*Fig.6. Scatter plot of 16 QAM with reduction of phase noise*



*Fig.8. Scatter plot of 16 QAM with phase noise variance of 1.0*



*Fig.9. Scatter plot of 16 QAM with phase noise (variance of 1.0) reduction*



*Fig.10. Bit error rate vs SNR plot at a phase noise variance of 1.0*



*Fig.12. Bit error rate vs SNR plot at a phase noise variance of 0.094*

### **CONCLUTION**

OFDM (Orthogonal Frequency Division Multiplexing) has been standardized in many recent wireless applications due to its ability to combat multipath effects, ISI and make better use of the system available bandwidth.

An exhaustive analysis of phase noise effects on OFDM signals has been presented in this paper. A computationally efficient technique is proposed here.

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